# Block Voting and the Electoral College

This is an addendum to “Three Fifths of a Floridian,” a review of the Electoral College Primer, by Longley and Pierce, http://www.ghg.net/redflame/florida.htm

48 of the 50 states use the "unit rule" in the Electoral College, which means that all of their electoral seats go to one candidate (using plurality voting). This is a an example of block voting. Let's consider the effects of this block voting in a simplified model of the US, with no Senate and no "safe" states. We'll consider the Senate and safe states later. Odd numbers are convenient to avoid ties in this example.

In order to make a meaningful comparison between the effects of small numbers of voters in large vs. small states, we need to use a consistent "unit of analysis," the number of voters we lump together in a group. One one extreme, we could treat each voter individually. On the other extreme, we could divide the total number of voters by the total number of electors. I have arbitrarily chosen to use 101 voting units per elector. For this first, simple model, I am treating each unit of voters as voting randomly, as if by tossing a coin (a binomial distribution). Unless we make the unit of analysis very large (small number of units), the probability of an election coming down to one unit will be very small and will be inversely proportional to the square root of the number of units (Stirling's formula). But I don't care about the actual numbers, only the ratio between them so I can compare large vs. small states. As long as this formula holds, I don't care what the unit of analysis is, only that it's consistent.

Here's my model:

Larry Large and Sally Small live in the new Republic of Slobovia, which has an Electoral College similar to that in the US. Slobovia has 51 states, 6 large ones with 35 House seats each, including Largistan, where Larry lives, and 45 small ones with 5 seats each, including Smallistan, where Sally lives, for a total of 435 Electors. (Slobovia has no Senate.) Slobovia is a very small country, with only 101 voters per House seat, but they are proud of being independent thinkers, so each and every one of them is a swing voter. If you want to predict how a Slobovian will vote, you might as well flip a coin. They have a two-party system (plurality voting), and each of the 51 states uses the "unit rule" whereby all of the state's Electors go to the candidate with the plurality of votes, no matter how slim.

What are the respective probabilities that the outcome of the Electoral College will come down to Larry's or Sally's vote?

For Sally to tip the election, there must be a tie among the other 5*101-1 = 504 voters in Smallistan, and the sum of the electoral votes for Sally's candidate from the other 50 states (6 large, 44 small) must be between 213 and 217.

For Larry to tip the election, there must be a tie among the other 35*101-1 = 3534 voters in Largistan, and the sum of the electoral votes for Larry's candidate from the other 50 states (5 large, 45 small) must be between 183 and 217.

The probability of Sally tipping Smallistan is the probability that the other 504 voters in Smallistan are tied. This would be the binomial distribution, where Sally's candidate wins 252 of 504 coin tosses at 50% probability each:

P_Sally_tips_local =

Binomial(x,n,P) = Binomial(252,504,0.5) = 0.03552.

The probability of Larry tipping Largistan is similarly:

P_Larry_tips_local = Binomial(1767,3534,0.5) = 0.01342.

The important thing here is that Largistan, with 7 times the number of voters as Smallistan, is less likely to be tipped by one vote (or any constant, small number of votes), by a factor of roughly the square root of 7. What we are really calculating here is the relative likelihood that a presidential candidate can win the state by pandering to a small (consistently sized) group of voters.

But Sally's tipping of Smallistan only makes a difference if Smallistan has enough Electors to tip the Electoral College. There are seven ways this can happen; seven possible combinations of the number of large states and the number of other small states that go for Sally's candidate and whose Electors add up to between 213 and 217 (218 votes are needed to win):

(#large,#small) =

(0,43), (1,36), (2,29), (3,22), (4,15), (5,8), or (6,1) .

For example, 0 large states * 35 votes / large state + 43 small states * 5 votes / small state = 215 votes, which is within 5 votes of the 218 needed to win.

If we use the shorthand B(x,n) = Binomial(x,n,0.5), the probability of this is

P_Small_tips_College =

B(0,6) * B(43,44) + B(1,6) * B(36,44) + B(2,6) * B(29,44)

+ B(3,6) * B(22,44) + B(4,6) * B(15,44) + B(5,6) * B(8,44)

+ B(6,6) * B(1,44)

= 0.01563 * 0.000 + 0.09375 * 0.00001 + 0.2344 * 0.01307

+ 0.3125 * 0.1196 + 0.2344 * 0.01307 + 0.09375 * 0.00001

+ 0.01563 * 0.000

= 0.04350 .

Similarly, there are six ranges of possible ways Largistan could tip the Electoral College to Larry's candidate. For each number of other large states that go Larry's way, there is a range of numbers of small states:

(#large,#small) =

(0,37-43), (1,30-36), (2,23-29), (3,16-22), (4,9-15), or (5,2-8) .

For example, two large states and between 23 and 29 small states give Larry's candidate between 185 and 215 votes, which are with 35 votes of the needed 218.

If we use the additional shorthand

C(xi,xf,n) = Cumulative Binomial(x=xi..xf; n,0.5),

the probability of Largistan tipping the Electoral College is

P_Large_tips_College =

B(0,5) * C(37,43,45) + B(1,6) * C(30,36,45) + B(2,6) * C(23,29,45)

+ B(3,5) * C(16,22,45) + B(4,5) * C(9,15,45) + B(5,5) * C(2,8,45)

= 0.03125 * 7.687E-6 + 0.1563 * 0.01784 + 0.3125 * 0.4822

+ 0.3125 * 0.4822 + 0.1563 * 0.01784 + 0.03125 * 7.687E-6

= 0.3069 .

The probability of Sally Small tipping the Electoral College is

P_Sally = P_Sally_tips_local * P_Small_tips_College

= 0.03552* 0.04350= 0.001545 .

The probability of Larry Large tipping the Electoral College is

P_Larry = P_Larry_tips_local * P_Large_tips_College

= 0.01342* 0.3069 = 0.004119 .

Larry is 0.004119 / 0.001545 = 2.665 times as likely as Sally is to tip the Electoral College.

For reference, if the President of Slobovia were elected by direct popular vote, the probability of it being decided by one vote would be n = 6 * 3535 + 45 * 505 = 43,935 ;

x = 0.5 * (n + 1) = 21,968 ;

Binomial(x,n,0.5) = 0.003807 .

Now redo this with two Senators per state (435 + 2 * 51 = 537 Electors). The 6 large states have 37 Electors each, and the 45 small states 7 each. It takes 269 Electors to win. The probabilities of Larry or Sally tipping their respective states are unchanged, but now for Largistan to tip the College, the Electors that go to Larry's candidate from the other 50 states must add up to between 232 and 268. For Sally, the corresponding numbers are 262 and 268. The 7 combinations of other respectively large and small states that work for Sally are:

(#large,#small) =

(0,38), (1,33), (2,27), (3,22), (4,17), (5,11), or (6,6) .

P_Small_tips_College (with Senate) =

B(0,6) * B(38,44) + B(1,6) * B(33,44) + B(2,6) * B(27,44)

+ B(3,6) * B(22,44) + B(4,6) * B(17,44) + B(5,6) * B(11,44)

+ B(6,6) * B(6,44)

= 0.01563 * 4.01E-7 + 0.09375 * 0.000434 + 0.2344 * 0.03901

+ 0.3125 * 0.1196 + 0.2344 * 0.03901 + 0.09375 * 0.000434

+ 0.01563 * 4.01E-7

= 0.05575 .

The combinations of other large and small states that allow Largistan to tip the Electoral College are:

(#large,#small) =

(0,34-38), (1,28-33), (2,23-27), (3,18-22), (4,12-17), or (5,7-11) .

P_Large_tips_College (with Senate) =

B(0,5) * C(34,38,45) + B(1,6) * C(28,33,45) + B(2,6) * C(23,27,45)

+ B(3,5) * C(18,22,45) + B(4,5) * C(12,17,45) + B(5,5) * C(7,11,45)

= 0.03125 * 0.0004118 + 0.1563 * 0.06717 + 0.3125 * 0.4324

+ 0.3125 * 0.4324 + 0.1563 * 0.06717 + 0.03125 * 0.000412

= 0.2913 .

The probability of Sally Small tipping the Electoral College (with Senate) is

P_Sally = P_Sally_tips_local * P_Small_tips_College

= 0.03552 * 0.05575 = 0.001980 .

The probability of Larry Large tipping the Electoral College (with Senate) is

P_Larry = P_Larry_tips_local * P_Large_tips_College

= 0.01342 * 0.2913 = 0.003909 .

Larry is 0.003909 / 0.001980 = 1.974 times as likely as Sally is to tip the Electoral College.

Block voting is also bad for reasons that were discussed in the section on implicit vs. explicit bargaining.

However, the point of the Slobovia example is to illustrate that people who claim that the Electoral College benefits the smaller states don't know what they're talking about.

A much more serious criticism of the Electoral College is that only some states are swing states, and political parties with presidential aspirations have enormous incentives to pander to swing voters in swing states at the expense of the rest of the country. Examples of this are the Clinton administration's handling of the Elian Gonzales incident and the Bush 43 administration's support for steel tariffs. Let's look at the effect that unbalanced party loyalties have on the likelihood of a presidential candidate being able to win a state by pandering to a few small groups of voters.

If the difficulty of winning over a group of voters were the same for each group of voters, the "cost" of winning over several equal-sized groups would be proportional to the number of groups needed. Imagine Karl Rove firing a cannon into a state, and if the cannonball (or "panderball") hits something, G. W. Bush gets 10,000 additional votes. Thus a "safe" Democratic state with a 10-percentage point Democratic advantage would be ten times as difficult (or unlikely, or expensive in terms of some sort of political "capital") for Bush to be able to win over as a "swing" state with a 1-percentage point Democratic advantage.

But if these "panderballs" are aimed, one would aim at the easiest targets first, and move to progressively harder targets as the easy targets are hit. If the difficulty of winning over a group of voters increases in linear proportion to how many easier groups have been won over, the cost of winning a state would vary approximately with the square of the other party's advantage. In this case, the 10-point Democratic state would be roughly 100 times as hard for Bush to win as a 1-point state.

Again, we could divide voters into 101 groups (or some other unit of analysis) and calculate the probability of winning as a function of how many of these were groups of swing voters, or were committed to one or the other party. For example, in a state with 30 groups for party A, 30 for party B, and 31 coin tossers, getting one group of coin tossers to commit to party A raises A's chance of winning from 0.5 to 0.562685 , a difference of 0.062685 . On the other hand, in a safe party B state, with 20 groups for A and 40 for B, getting one group of swing voters to commit only raises the odds from 0.000725246 to 0.00111072, a difference of 0.00038547 . Thus a persuadable group of voters in this swing state is worth 162.62 times as much political capital as in the safe state. Increasing a lead in an already safe state is similarly pointless in this model.

Case 1:
safe n = 101-30-30 = 41
needed x = 0.5*(101+1)-30 = 21

Case 2:
safe n = 101-30-31 = 40
needed x = 0.5*(101+1)-31 = 20

Case 3:
safe n = 101-40-20 = 41
needed x = 0.5*(101+1)-20 = 31

Case 4:
safe n = 101-40-21 = 40
needed x = 0.5*(101+1)-21 = 30

Cumulative Binomial(x=21..41,n=41,P=0.5) = 0.5
Cumulative Binomial(x=20..40,n=40,P=0.5) = 0.562685
Cumulative Binomial(x=31..41,n=41,P=0.5) = 0.000725
Cumulative Binomial(x=30..40,n=40,P=0.5) = 0.001111

This model may be useful for illustration purposes, but it is sensitive to the unit of analysis. With a very small unit of analysis (large number of voters or groups of voters), the odds are astronomically in favor of the swing voters (coin tossers) being nearly evenly split.

Proponents of the Electoral College often claim that it gives a candidate an incentive to work towards uniform support throughout the country, working hardest for support in areas where he is weakest. In fact, neither candidate has an incentive to campaign hard in any area where they do not each already have close to 50% support.